A Review of Instructional Goals and Strategies to Enhance Essentials of Mathematics Communication in Schools

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Abstract

Mathematics is still dealt as content only in most schools. This is despite increasing realization of it as a language. Approaching mathematics as a language calls for teaching and learning it with intent to develop skills to listen, speak, read and write mathematics and to get students acquainted with mathematical registers and discourse forms. In order to help teachers and researchers to strengthen mathematical language skills in school students, focal objectives, instructional strategies, classroom environment and role of teacher in enhancing essentials of mathematics communication in schools are drawn and summarized from a review of related literature. Inter alia, a communicative approach to teaching and learning mathematics is also discussed

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Teachers and educators increasingly recognize Mathematics as a language. Though mathematics is not identical with a natural language, it also requires oral and written communication with its own vocabulary, symbols, and concepts, rules and conventions. Listening to, comprehending and speaking math also require special skills. Thus communication in and about mathematics needs to be an important part of learning mathematics in schools. Facilitating precision in communication is especially important in mathematics. Learning the skills of communication in mathematics, for instance, is essential for solving problems, understanding how others solve mathematics problems, and to work with others to solve challenging problems. However, developing language skills in mathematics like listening, speaking, writing and speaking, though not to same extent, are largely neglected in teaching learning mathematics in schools, globally, but especially in multilingual classrooms in countries like India. Hence, for the sake of supporting the emphasis on mathematics communication and acquisition of related language skills in school, this paper through a review of literature available globally, attempts to bring together language related objectives of teaching mathematics, and steps that teachers can take to create and adopt appropriate environment and strategies to implement mathematics language skill in a schools.

Objective of this Review

To identify instructional goals and suggested instructional strategies /techniques from existing literature in relation to LSRW skills in mathematics at school level.

Method

Review of 23 documents including books, book chapters, research articles and teacher resources that provided methods, techniques, and strategies to develop LSRW skills in mathematics. This review restricted to listening, speaking, reading and writing as the mathematics language skills though mathematical language encompasses the other language functions and vocabulary.

Observations from Review

The Focus: Why Emphasize the Mathematics Communication Skills in Classrooms?

A variety of explicit and implicit, immediate as well as long term outcomes of instructional practices that purposefully provide students opportunities to listen speak read
and write and discourse in mathematics are recognized in literature. Obvious direct effects of including mathematics language skills in instruction is that students acquire ability to listen to, speak, read and write mathematics and acquire the ability to engage in mathematics discourses (NALDIC, 2002). Beyond those immediate outcomes, exercising and developing the skills in listening is proposed to help learners to become critical thinkers about mathematics (NCTM, 2000) and to negotiate the symbols, diagrams, and technical language (Schleppegrell, 2007). Likewise, speaking mathematics give students opportunity to explain, develop and name mathematical theories, and promote greater clarity in their thinking and verbalization (Lee, 1997). Speaking with current understanding of mathematical ideas enables learners to become aware of, develop and reorganize their knowledge; to remember what they have worked with, and makes the knowledge available for them to use and control. Essentially, by speaking mathematics and involving in mathematical discourse, students learn to assign meanings to words and phrases which are shared within a community, learn mathematical concepts, and turn out to be self-confident of solving mathematical problems (Lee, 2006).

Through reading mathematics, students learn to recognize and comprehend terminology, numbers, mathematical symbols and expressions, formulae, charts, diagrams, tables and graphs. This in turn facilitates understanding the technical language of mathematics, patterns and relationships in mathematical problems. Reading and comprehending these various forms of mathematics representations essentially helps them to consolidate learning in mathematics (SCAA, 1997 as in NALDIC, 2002). Redraft presented information or learning to prepare a mathematical argument or justification or formulate conclusions (NALDIC, 2002) are valuable mathematics learning outcomes in themselves. Writing help learners to be metacognitive by reflecting on their work, clarifying their thoughts about the ideas, and rereading the record of their own thoughts (NCTM, 2000).

Discourses in mathematics classrooms apart from being helpful for teachers to evaluate students’ ability to use technical language appropriately, develops in students a register of technical language of mathematics that in turn enables them to develop connections between the everyday meanings of words and their mathematical meanings (Schleppegrell, 2007). Posing rich Open-ended and challenging tasks that promote discussions in classrooms encourage students to think collaboratively and build upon one another’s ideas (Stein, Smith, Henningsen, & Silver, 2000 as in Clark, Jacobs, Pittman & Borko, 2005) which further enhances such mathematical discourse (Lee, 2006).

Strategies to Enhance Mathematics Communication in Classrooms

Listening to Teachers and Peers

Listening can be encouraged by teacher explanations that use mathematical language and vocabulary as well as non-mathematical explanations of mathematical ideas; and teacher and pupils exploring mathematical processes, reasoning and proving the solutions to a problem (NALDIC, 2002). Students need to listen closely to the thinking of others, take their ideas seriously." (p. 349). One aspect of taking students’ ideas seriously is ensuring that their classmates attend to the ideas and work to understand them. This requires also that classroom activities should be structured to ensure that students have ample time and encouragement to process others’ ideas (Clark, Jacobs, Pittman & Borko, 2005).

Developing Speaking Skills in Mathematics

Inquiry environments and questioning

Skills in speaking mathematics require an inquiry environment in the mathematics classroom. This involves inviting students to share their strategies, pose questions, and “think out loud” (Cobb, Wood, Yackel, & McNeal, 1992; Grouws & Cebulla, 2000 as in Clark, Jacobs, Pittman & Borko, 2005). Effective questioning of pupils stimulates an inquiry environment. Give students time to think; and expect them to demonstrate and explain their reasoning.
Another way is to explore reasons for wrong answers if any. Students can also be required to explore mathematical concepts; describe shapes, movements and constructions; explain calculation strategies and methods for solving problems; reason solution plans and justify results; compare different efficiency and effectiveness of different mathematical procedures; discuss which mathematical equipment and materials to use; and present their findings to an audience (NALDIC, 2002). Students need be involved orally and in writing in explaining solution processes, describing conjectures, proving conclusions and presenting arguments (Schleppegrell, 2007). Teachers have to model how to talk formally, for instance, by using numbers or symbols instead of the pronouns and demonstratives (Pimm, 1987 as in Schleppegrell, 2007). NCTM (2000) also suggests formulating questions that puzzles students, then students presenting their methods for solving problems and justifying their reasoning to their peers and teachers in a coherent and clear manner as facilitating speaking in mathematics. Student-led discussions regarding the problems of the day and allowing for agreement and disagreement from others will encourage thoughts and interactions (Sample, 2009).

**Think aloud**

An important means to develop speaking skill is to create rich opportunities for students to explain their thinking (Barwell, n.d.). Especially, children in the early grades may be made to “think out loud,” to learn to explain their answers and describe their strategies through thoughtful questions that provoke reasoning. Such questions provoking them to reexamine their reasoning can be posed by a teacher or classmate, and as they gain experience of think aloud procedures, student proficiency in organizing and recording their thinking is enhanced. (NCTM, 2000). By presenting their methods for solving problems, and giving verbal accounts and explanations, students gain insights into own thinking (Silver, Kilpatrick & Schlesinger 1990 as in NCTM, 2000). “Think aloud” is possible even while explaining their solution strategies for multiple-choice and short answer items (Capraro & Joffrion, 2006). Working out arithmetic aloud has other advantages too. This procedure allows the group to know what other members are doing and to check their own answer (Lee, n. d). Other methods to enhance thinking and speaking in mathematics advocated by Lee include stating and restating the problem, vocalizing the arithmetical workings, and challenging others’ observations and providing answers when challenged and revealing when they feel uncertain about the solution to a problem. Interacting in such a way that give access to reasoning of others scaffold mathematical thinking-speaking.

**Code switching**

Code switching is a most frequently used technique to connect everyday language and mathematics language. This involves alternation in use of more than one language in a single speech act. Code-switching can be between languages or between the mathematical and everyday registers. Teachers can switch codes in order to translate or clarify instructions and to reformulate and model appropriate mathematical language use. Students can be allowed to switch codes to seek clarification and to express their ideas or arguments (Zazkis, 2000).

**Developing Reading Skills in Mathematics**

**Nine-stage instruction model**

Reading skills can be developed in mathematics problem solving situations. Nine-stage instruction model developed by Ilany and Margolin (2010) utilizes reading as a means for problem solving. This involves, among others, reading the problem repeatedly. (1) Initial reading is from words to the whole text as a way of collecting details and understanding meaning. (2) Second reading is for understanding the keywords, sentence and describing the problem in own words. (3) Understanding the Mathematical Situation through explicit or implicit the data in the problem. Processing the literal information and changing it into a mathematical exercise or an algebraic equation is done using the literal clues like words that
support (helpful clues) or the words that deceive (misleading clues) as clues for choosing the arithmetic operations needed to solve the problem.

**Reading word problems**

Another way to encourage reading using mathematics problems is to read word problems out loud, elaborating and commenting on what it says that encourage students to talk about the meanings (Adams, 2003; Chapman, 1995; Lemke, 1989 as in Schleppegrell, 2007). Meaningful reading can be facilitated via equipping learners to be strategic readers. Teachers can model the strategic reading process by reading the problem out loud (Metsisto as in kenney, 2005). Strategic readers Preview the text, look for title, the pictures, and thereby activate appropriate prior knowledge, the vocabulary, and clarify their purposes for reading by asking questions on what to learn. While reading, strategic learners paraphrase the author’s words, monitor comprehension, use context clues to figure out unknown words, and imagine, infer, and predict in and out of text to integrate new concepts with existing knowledge. Teachers can model raising questions relevant to mathematics problem (Metsisto as in kenney, 2005).

**Reading books in mathematics**

Reading books in mathematics can emphasize quantitative language (e.g., more, fewer, a lot, a little) and spatial language (e.g., higher, lower, above, below, before, after) wherever possible by incorporating questions that asks for responses in such language. Such prompting questions in reading books need be increasingly more complex to make children familiar with mathematical content and language before advancing to more in-depth prompts and discussions including distancing strategies. In using such reading books teachers can also ask questions that call for spontaneous responses and feedback in classroom to reinforce children’s understanding of the mathematical language used. Teachers may even explicitly define or explain mathematical language terms if their class demonstrates a lack of understanding of the terms involved in the questions or text. However, teachers need not explicitly define each word for the whole class. This will be achieved by learners through the context of the pictures, text, and questions. Such dialogic reading frameworks are known to use strategies like PEER (Prompt, Evaluate, Expand, and Repeat) strategy and CROWD (Completion, Recall, Open-ended, Wh- [what, where, why) during readings. Note cards placed at specific points in text book can also prompt dialogic reading (Purpura, Napoli, Wehrspann & Gold, 2016).

**Developing Writing Skills in Mathematics**

**Exposure to writing authentic genres in Mathematics**

Though students write a lot while learning mathematics, student exposure to writing different genres of mathematics is often limited. Mathematics communication requires giving practice in writing authentic genres of mathematics like presentation of procedures, descriptions and classes of things, explanations of judgments or findings, and arguments about theorems and other mathematical tasks (Schleppegrell, 2007) and correcting someone else’s mathematical writing (Lee, 2006). Students need be provided with a wide variety of writing samples relating to mathematics (Ryder & Graves 2003 as in Brummer & Clark, 2013). In this context, a dynamic and live bulletin board in the classroom that displays newer writing samples adds to curiosity about the new additions. (Brummer & Clark, 2013).

**Common approaches**

A variety of common sense approaches like providing plenty of time for students to experience the writing process, Offering daily writing opportunities, Allowing time for students to evaluate others’ writing and receive teacher feedback, Encouraging learning new mathematics words, Focusing on students’ reading and writing on big ideas, providing opportunities to read, understand, and write about increasingly complex text (Corona, Spangengerber, & Venet 1998 as in Brummer & Clark, 2013). Sample (2009) for example,
provided approximately 10 minutes each day to give each students time to write out their solutions to the problem of the day.

Structured strategies

There are many structured strategies that support, and direct students writing on specific mathematics learning related tasks. In Composing with Key words, students compose mathematically related sentences and paragraphs with words from Taxonomies (Rothstein, Rothstein & Lauber, 2006). Another specific strategy is Defining Format where a three-column format defines a mathematical term (e.g., number), using a question, the category, and the characteristics. Yet another structured strategy is using profiles and frames. Profiles are templates into which students plug appropriate information to solve a problem or explain a mathematical concept. In Frames, however, the students are given the syntactic structure, which includes stem or partial sentences. Completing these partial sentences helps students focus on the content of their writing without concern for grammatical or structural aspects of the text. (Rothstein, Rothstein, & Lauber, 2006). Worksheets with title, the learning target, concept notes with illustrative examples and vocabulary of key terms will also be useful (Rothstein, Rothstein, & Lauber, 2006).

Reflection writing

Students can also be asked to write their reflection on what they learned, quantity of work done during the lesson and to identify topics they did not understand and have found difficult (Lomibao, Luna & Namoco, 2016). Such Metacognitive Frames enhances Self-awareness of mathematical knowledge by making students write statements such starter phrases that you must complete, I know that I know something about..., First I know; In addition, I know; ‘Finally, I know’; ‘Now you know something that I know’.

Developing Discourse Skills in Mathematics

Think, talk, write, read and re-draft

Think, talk, write, read and re-draft strategy by Lee (2006) incorporates all four language skills into a single strategy. Strategy elements are as follows. 1) Give time to think quietly by themselves for a few seconds and ask them to write five words that they associate with a concept in a whole-class questioning session. 2) discuss their ideas with one or more partners (‘response partners’ or ‘study buddies’) and make some decisions for between 30 seconds and a few minutes for an answer with a minimum length, depending on the question; rehearsing the answer helps pupils feel confident to add their contributions to a whole-class mind-map or spider diagram. 3) Read to themselves and to other people and Re-draft based on feedback received, to improve their work and to produce high-quality communications and 4) write only when they have thought and talked about their ideas. Display pupils writings for everyone and ask them to consider which wording or phrasing expresses the ideas most clearly to help pupils towards fluency with mathematical language.

RPTMC Activity flow

Yang, Chang, Cheng and Chan, (2016) developed Reciprocal peer tutoring-enhanced mathematical communication activity flow (RPTMC Activity flow) where every two student were paired as a mathematical communication group. Four sub activities are Creating, Reciprocal peer tutoring, revising and staging,

1. Creating: required students to prepare tutoring materials involved four steps. 1). Understanding the problem where Students read the word problem on their own tablet PCs and discuss the solution with their peers to understand the conditions given and the problem asked. 2) Drawing a representation where Students use words, symbols, models, and manipulative materials as their mathematical representations to devise a plan as well as to convey their ideas and communicate information. 3) Writing a solution where Students write their mathematical equations for solving the problem and 4)
Explaining the solution were Students reflected on how and why they had solved the problem and explained their solution in writing. Because students may need guidance in learning how to express their mathematical concepts before they could write a complete sentence explaining their solutions, a text-based scaffold was provided.

2. Reciprocal peer tutoring: Paired students sat together to reciprocally teach their mathematics creations. One student, who played the role of a tutor, taught his/her peer why and how to solve word problem by displaying mathematics creation in the Sharing Zone (designed for easy display of mathematics creations). While the other student who played a tutee, received instruction with the tutor’s mathematics creation in his/ her own tablet PC. Subsequently the tutee has to ask questions about solution strategy. The paired students then switch their roles.

3. Revising: students had to revise their mathematics creation based on the peer feedback in the previous activity for improving the clarity of their own mathematics creations. Revising also served as a time for self-reflection and preparation for the next activity staging.

4. Staging: Teacher encouraged the students in each group to display their mathematics creation of the whole class. They had to explain their solution with their representations to the audience. Then they had to answer questions asked by the audience. In the end teacher used students work to demonstrate how to explain the mathematics concepts and to clarify mistakes made by students for preventing similar ones in future. Moreover teacher may ask some relevant questions to promote students’ thinking for communicating their own mathematical concepts and thinking with others.

Other discourse strategies

Other methods found in literature for encouraging discourse in mathematics classrooms include open-ended problem/questions (Lomibao, Luna & Namoco, 2016; Wichelt, 2009), Revoicing (technique for interaction) (Anthony & Walshaw, 2009; Moschkovich, 1999 as in Schleppegrell, 2007; Chapin, Connor & Anderson, 2009), leading mathematical conversations (Patkin, 2011), activity sheets, and classroom discussions (Schleppegrell, 2007).
Conclusion

The review of available literature shed light on some relevant, immediate as well as long term goals of mathematics teaching when one considers mathematics as a language. Yet, these goals are generally neglected in schools. An emphasis on language of mathematics and communicating through mathematics language in teaching engenders learning that is more meaningful, and conceptually integrated. More reading brings in better consolidated learning in mathematics. Discourses in mathematics help to develop a register of technical language of mathematics, making connections between everyday meanings of words and their mathematical meanings possible for students. Discourse driven mathematics classrooms are helpful for teachers as well, as they reveal students’ ability in the area being discussed.

This review has brought together an array of strategies suitable for developing listening, speaking, reading and writing skills in mathematics among school students. Attributes of classroom strategies that enhances student acquisition of mathematics language skills include teacher explanations, inquiry environment, oral and writing participation, modeling and scaffolding by teachers on the intended skills, formulating questions/puzzles that engenders discussion, opportunities for students to explain their thinking, code switching, strategic reading of problems, opportunity in writing authentic genres, reflective writing on mathematics learning along with common sense approaches and specifically structured strategies.

Obviously, integration of language into mathematics classroom will develop student skill in using technical language appropriately. Paying attention to the language of mathematics in classrooms, apart from the acquisition of listening speaking reading and writing skills, help learners; to become aware of, recognize, develop and reorganize their knowledge, to negotiate the language, to articulate their understanding, to consolidate their learning, to develop critical thinking about mathematics, to develop connections between mathematics and life, to think collaboratively and to build upon one another’s ideas and to increasingly engage in mathematical discourses. Apart from structured and specific instructional procedures, instruction focusing on language of mathematics frequently make use of exploring
mathematical processes, talking, questioning, stating and restating problems and uncertainties, reasoning, thinking aloud, challenging others’ observations and providing answers, building explanations, and justifying and the like in whole class and varied group environments. This in turn will help them to get greater clarity in their thinking and verbalization, and hence in mathematics communication. Improved communication among peers and with teachers turn learners to meet cognitively and critically reflect on mathematics. Hence, an increased emphasis on communication through language of mathematics in schools will bring in for their students deeper engagement and understanding, greater independence and self-regulation, and, stronger competence with mathematical processes.

Reference


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